

Precalc 12 Mid-Year Review Answers

1a) $(f+g)(x) = f(x) + g(x)$
 $= 9x^2 - 1 + 3x - 1$
 $= 9x^2 + 3x - 2 //$

b) $(f-g)(x) = f(x) - g(x)$
 $= 9x^2 - 1 - (3x - 1)$
 $= 9x^2 - 1 - 3x + 1$
 $= 9x^2 - 3x //$

c) $(fg)(x) = f(x)g(x)$
 $= (9x^2 - 1)(3x - 1)$
 $= 27x^3 - 9x^2 - 3x + 1 //$

d) $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{9x^2 - 1}{3x - 1} \quad x \neq \frac{1}{3}$
 $= \frac{(3x+1)(3x-1)}{3x-1} = 3x+1 \quad (x \neq \frac{1}{3})$

1B a) $f+g = \frac{x}{x-2} + \frac{3}{x}$
 $x \neq 2, 0$
 $= \frac{x^2 + 3(x-2)}{x(x-2)}$
 $= \frac{x^2 + 3x - 6}{x(x-2)} //$

b) $f-g = \frac{x}{x-2} - \frac{3}{x}$
 $= \frac{x^2 - 3(x-2)}{x(x-2)}$
 $= \frac{x^2 - 3x + 6}{x(x-2)} //$
 $x \neq 0, 2$

c) $fg = \frac{x}{x-2} \cdot \frac{3}{x} \quad x \neq 0, 2$
 $= \frac{3}{x-2} //$

d) $\frac{f}{g} = \frac{x}{x-2} \div \frac{3}{x} = \frac{x}{x-2} \cdot \frac{x}{3} = \frac{x^2}{3(x-2)} //$
 $x \neq 0, 2$

a 2) $f+3g = (x^2-1) + 3(3-2x)$
 $= x^2 - 1 + 9 - 6x$
 $= x^2 - 6x + 8$
 $= (x-4)(x-2) //$

b) $2f - fg = 2(x^2-1) - (x^2-1)(3-2x)$
 $= 2x^2 - 2 - (3x^2 - 2x^3 - 3 + 2x)$
 $= 2x^2 - 2 - 3x^2 + 2x^3 + 3 - 2x$
 $= 2x^3 - x^2 - 2x + 1 //$

a 3)

| | | |
|----|------|----|
| x | f(x) | |
| 4 | → | 5 |
| -2 | → | -1 |
| 3 | → | 4 |
| -1 | → | 0 |

g(f(x))
 \emptyset

| | | |
|------|------|---------|
| b) x | g(x) | g(g(x)) |
| -2 | → | -1 |
| -4 | → | -2 → -1 |
| 6 | → | 3 |
| 8 | → | 4 |

∴ g(g(x)) = -1

$$4c) f(x) = \sqrt{x+2} \quad D: x \geq -2 \quad R: y \geq 0 \quad g(x) = 3x \quad D: \mathbb{R} \quad R: \mathbb{R}$$

$$f \circ g = \frac{\sqrt{3x+2}}{\sqrt{3x+2}} \quad D: x \geq -\frac{2}{3} \quad R: y \geq 0 \quad g \circ g = 3(3x) = 9x // \quad f \circ f(-3) = \sqrt{\sqrt{-3+2} + 2} = \emptyset$$

$$g \circ f = 3\sqrt{x+2} \quad D: x \geq -2 \quad R: y \geq 0$$

$$4d) f(x) = 3x+1 \quad D: \mathbb{R} \quad R: \mathbb{R} \quad g(x) = |-x| \quad D: \mathbb{R} \quad R: y \geq 0$$

$$f \circ g = 3|-x|+1 \quad D: \mathbb{R} \quad R: y \geq 1 \quad g \circ g = |-|-x||$$

$$g \circ f = |-(3x+1)| = |-3x-1| \quad D: \mathbb{R} \quad R: y \geq 0$$

$$f \circ f(-3) = 3(3(-3)+1)+1 = 3(-8)+1 = -23 //$$

$$4e) h(x) = \frac{x}{x-1} \quad D: x \neq 1 \quad R: y \neq 0 \quad k(x) = \frac{1}{x^2} \quad D: x \neq 0 \quad R: y \neq 0$$

$$h \circ k = \frac{\frac{1}{x^2}}{\frac{1}{x^2}-1} = \frac{\frac{1}{x^2}}{\frac{1-x^2}{x^2}} = \frac{1}{x^2} \cdot \frac{x^2}{1-x^2} = \frac{1}{1-x^2} // \quad D: x \neq \pm 1 \quad R: y \neq 0$$

$$k \circ h = \frac{1}{\left(\frac{x}{x-1}\right)^2} = \frac{(x-1)^2}{x^2} // \quad D: x \neq 0 \quad R: y \geq 0$$

$$k \circ k = \frac{1}{\left(\frac{1}{x^2}\right)^2} = x^4 // \quad h \circ h(x) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$$

$$h \circ h(-3) = \frac{\frac{-3}{-3-1}}{\frac{-3}{-3-1}-1} = \frac{\frac{-3}{-4}}{\frac{-3}{-4}-1} = \frac{\frac{3}{4}}{\frac{3}{4}-1} = \frac{3}{4} \div \frac{-1}{4} = -3 //$$

a) $f(x) = 4x + 2$
 $x = 4y + 2$
 $y = \frac{x-2}{4}$

check $f \circ f^{-1} = 4\left(\frac{x-2}{4}\right) + 2$
 $= x - 2 + 2$
 $= x //$

b) $f(x) = x^2 - 5$
 $x = y^2 - 5$
 $x + 5 = y^2$
 $y = \pm\sqrt{x+5}$

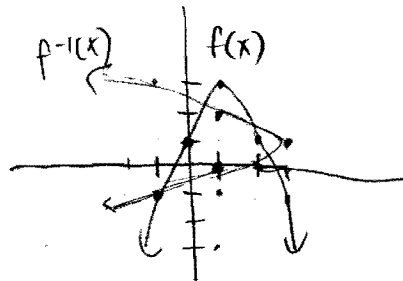
check $f \circ f^{-1} = (\pm\sqrt{x+5})^2 - 5$
 $= x + 5 - 5$
 $= x //$

c) $g(x) = \frac{2}{3-x}$
 $x = \frac{2}{3-y}$
 $3-y = \frac{2}{x}$
 $y = 3 - \frac{2}{x}$

$g \circ g^{-1} = \frac{2}{3 - (3 - \frac{2}{x})}$
 $= \frac{2}{\frac{2}{x}}$
 $= x //$

a) $f(x) = 3 - 2(x-1)^2$
 $x = 3 - 2(y-1)^2$
 $\frac{x-3}{-2} = (y-1)^2$
 $\pm\sqrt{\frac{3-x}{2}} = y-1$
 $y = 1 \pm \sqrt{\frac{3-x}{2}}$

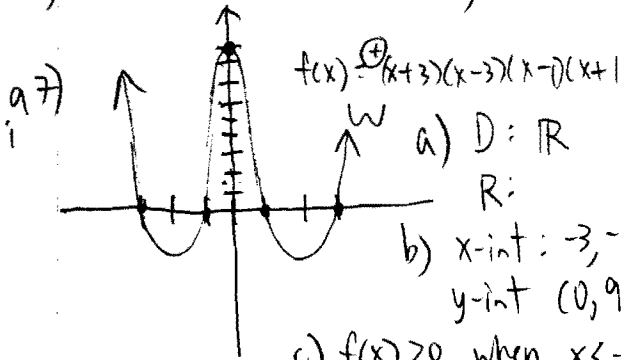
b) $f(x) = -2(x-1)^2 + 3$



| | | |
|-------------|--------------|--------------|
| | D | R |
| $f(x)$ | \mathbb{R} | $y \leq 3$ |
| $f^{-1}(x)$ | $x \leq 3$ | \mathbb{R} |

c) $x \geq 1$ or $x < 1$

d) bleh



a) $D: \mathbb{R}$
 $R:$

b) x -int: $-3, -1, 1, 3$
 y -int: $(0, 9)$

c) $f(x) > 0$ when $x < -3$
 $-1 < x < 1$
 $x > 3$

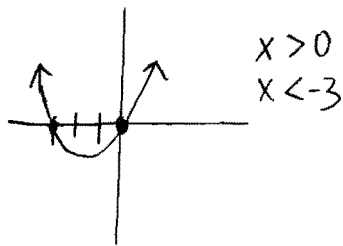
$f(x) < 0$ when $-3 < x < -1$
 $1 < x < 3$

d) symmetry at $x=0$; both ends increasing

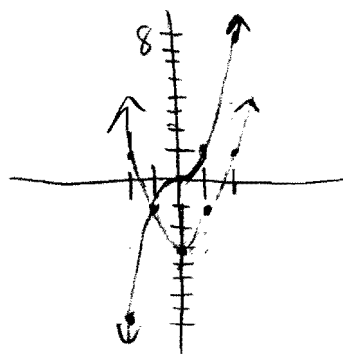
7a) $y = x^5 + 3x^2 - 2$

ii) need a graphing program.

a8) $x(x+3) > 0$



b) $x^3 < 2x^2 - 3$



$x < -1$

9) Throw these into a graphing program

i) $f(x) = \frac{2}{3-x}$

VA @ $x=3$
HA @ $y=0$
D: $x \neq 3$
R: $y \neq 0$

ii) $g(x) = \frac{x}{x+4}$

VA @ $x=-4$
HA @ $y=1$
D: $x \neq -4$
R: $y \neq 1$

iii) $f(m) = \frac{m^2-9}{m-3} = \frac{(m+3)(m-3)}{(m-3)}$

no VA or HA
hole @ $m=3$
D: $m \neq 3$
R: $y \neq 6$

iv) $f(p) = \frac{p^2-2p-8}{p+2} = \frac{(p-4)(p+2)}{p+2}$

no VA or HA, hole @ $p=-2$
D: $p \neq -2$
R: $y \neq -6$

a 10) $y = 2\sqrt{x+2} - 4$
D: $x \geq -2$
R: $y \geq -4$

b) $y = -\sqrt{x-1} + 3$
D: $x \geq 1$
R: $y \leq 3$

a 11) $\sqrt{x+1} + 2 = 6$
 $\sqrt{x+1} = 4$
 $x+1 = 16$
 $x = 15$

check: $\sqrt{15+1} + 2 = 6$
 $6 = 6 \checkmark$

$x = 15$

b) $m = -\sqrt{m+6}$
 $m^2 = m+6$
 $m^2 - m - 6 = 0$
 $(m-3)(m+2) = 0$
 $m = 3, -2$

check $m=3$
 $3 \stackrel{?}{=} -\sqrt{3+6}$
 $3 \neq -3$

reject $m=3$
check $m=-2$
 $-2 \stackrel{?}{=} -\sqrt{-2+6}$
 $-2 = -2 \checkmark$

$m = -2$

c) $\sqrt{x+3} = \sqrt{2x+1}$
 $x+3 = 2x+1$
 $2 = x$

check: $\sqrt{2+3} \stackrel{?}{=} \sqrt{2(2)+1}$
 $\sqrt{5} = \sqrt{5} \checkmark$

$x = 2$

d) $\sqrt{a+1} + 1 = \sqrt{a+3}$
 $(\sqrt{a+1} + 1)^2 = (\sqrt{a+3})^2$
 $a+1 + 2\sqrt{a+1} + 1 = a+3$
 $2\sqrt{a+1} = 1$
 $\sqrt{a+1} = \left(\frac{1}{2}\right)^2$
 $a+1 = \frac{1}{4}$
 $a = -\frac{3}{4}$

check: $\sqrt{-\frac{3}{4}+1} + 1 \stackrel{?}{=} \sqrt{-\frac{3}{4}+3}$
 $\frac{1}{2} + 1 \stackrel{?}{=} \sqrt{\frac{9}{4}}$
 $\frac{3}{2} = \frac{3}{2} \checkmark$

$a = -\frac{3}{4}$

ANSWER KEY

12) 22

13) 19

14) 2

15) 6, -11

16) -20

17) -6, -1, 3

18) $0, \frac{1}{2}, 1, 3$

19) -4, -2, -1, 3

20) $2 \times 3 \times 7$

21) $3 \times 3 \times 15$

22) a) 11 b) $\frac{7}{2}$

23) a) 81 b) 8 c) $\frac{1}{4}$ d) 6 e) $10^{\frac{23}{3}}$ f) 5 g) 9 h) $\frac{1}{3}$

i) $\frac{1}{2\sqrt{2}}$ j) $\frac{1}{2}$ k) $-\frac{1}{6}$ l) 9 m) 64

24) a) 1 b) $-\log_y x$ c) 2 d) $M^2 + M$

25) a) $x=1.465$ b) $x=2.269$ c) 2.795 d) -0.432

e) -5.095

26) a) domain: $x \in \mathbb{R}$ range: $y > 1$

b) domain: $x > 3$ range: $y \in \mathbb{R}$

c) domain: $\frac{3}{2} < x < 5, x \neq 2$ range: $y \in \mathbb{R}$

28) 11

29) $\frac{7}{2}x + 3$

30) $6y + x + 1$

31) 21.12 m

32) 23.44 weeks

33) 8.1

34) 7.87 days

35) 17.95 years