

## Operations with Functions

$$\textcircled{1} f(x) + g(x) = (3x-1)^2 + (x+2)^2 = 10x^2 - 2x + 5$$

$$f(x) - g(x) = 8x^2 - 10x - 3$$

$$f(x) \div g(x) = \frac{(3x-1)^2}{(x+2)^2}, x \neq -2$$

$$\textcircled{2} g(f(3)) = g(11) = 2 \cdot 11 - 5 = 22 - 5 = 17$$

$$\textcircled{3} f(g(x)) = (3x-1)^2 - 4(3x-1) = 9x^2 - 18x + 5$$

$$\textcircled{4} g(g(-\frac{2}{3})) = g(2 + \frac{10}{3}) = g(\frac{16}{3}) = 2 - 5 \cdot \frac{16}{3} = \frac{6-80}{3} = -\frac{74}{3}$$

$$\textcircled{5} f(g(x)) = \sqrt{6(2x+3)} = \sqrt{12x+18}$$

$$\textcircled{6} f(g(x)) = \sqrt{x+1} - 2; x \geq -1$$

$$\textcircled{7} A(r) = \pi r^2 \quad A(c) = \pi \left(\frac{c}{2\pi}\right)^2 = \pi \cdot \frac{c^2}{4\pi^2} = \frac{c^2}{4\pi}$$
$$r(c) = \frac{c}{2\pi}$$

## CH. 1.4-1.6 Transformations of Functions

$$\textcircled{1} y \mapsto -y \quad \textcircled{b}$$

$$\textcircled{2} \textcircled{b}$$

$$\textcircled{3} (a, b) \rightarrow \boxed{2}$$

$\swarrow$   
 $x \cdot \frac{1}{3}$

$$\textcircled{4} y = f(-(x-1)) + 3$$

$$(-2, 5) \rightarrow (2, -5) \rightarrow (3, -5) \rightarrow (3, -2)$$

$$\textcircled{5} \text{a) } y = \frac{1}{2} f[-(x-1)] - 2 \quad \text{b) } y = -f[2(x-1)] + 1$$

$$(6) \quad y \mapsto y-4 \quad 2x + 3(y-4) = 5$$

$$(7) \quad (2, -8) \rightarrow (5, -8) \rightarrow (5, -4)$$

$$(8) \quad x = \frac{y}{3y-1} \quad y(3x-1) = x$$

$$x(3y-1) = y \quad y = \frac{x}{3x-1}$$

$$3xy - x = y \quad f^{-1}(x) = \frac{x}{3x-1}$$

$$3xy - y = x$$

$$(9) \quad y = \frac{2x}{1-x} \quad x = xy + 2y$$

$$x = y(x+2)$$

$$x = \frac{2y}{1-y} \quad y = \frac{x}{x+2}$$

$$x - xy = 2y \quad f^{-1}(x) = \frac{x}{x+2}$$

$$(10) \quad y = f(2-x) = f[-(x-2)]$$

$$(-3, 0) \rightarrow (3, 0) \rightarrow (5, 0)$$

$$(0, 0) \rightarrow (0, 0) \rightarrow (2, 0)$$

$$(2, 0) \rightarrow (-2, 0) \rightarrow (0, 0)$$

$$(11) \quad y = \frac{1}{2} f(-(x+1)) - 2$$

$$(3, -4) \rightarrow (3, -2) \rightarrow (-3, -2) \rightarrow (-2, -2) \rightarrow (-2, -4)$$

$$(12) \quad (m, n) \rightarrow (m, -n) \rightarrow \left(\frac{m}{4}, -n\right) \rightarrow \left(\frac{m}{4} + \frac{1}{2}, -n\right) \rightarrow \left(\frac{2+m}{4}, -n+1\right)$$

$$(13) \quad x \rightarrow \frac{1}{3}x \quad \left(\frac{x}{3}\right)^2 + (2y)^2 = 1$$

$$y \rightarrow 2y$$

# Ch. 3 Radical Functions and Equations

① a)  $\sqrt{x-2} = 2 \quad ; x \geq 2$   
 $x-2 = 4$   
 $x = 6$

Check:  $\sqrt{6-2} = 2$   
 $2 = 2 \checkmark$

$S = \{6\}$

b)  $\sqrt{5x-6} = x$   
 $5x-6 = x^2$   
 $x^2 - 5x + 6 = 0$   
 $(x-2)(x-3) = 0$   
 $x = 2 \text{ or } x = 3$

Check:  $\sqrt{5 \cdot 2 - 6} = 2$   
 $2 = 2 \checkmark$   
 $\sqrt{5 \cdot 3 - 6} = 3$   
 $3 = 3 \checkmark$

$S = \{2, 3\}$

c)  $\sqrt{13-x} = x-1$   
 $13-x = x^2 - 2x + 1$   
 $x^2 - x - 12 = 0$   
 $(x-4)(x+3) = 0$   
 $x = 4 \text{ or } x = -3$

Check:  $\sqrt{13-4} = 4-1$   
 $3 = 3 \checkmark$   
 $\sqrt{13+3} = -3-1 \quad \times$

$S = \{4\}$

d)  $\sqrt[3]{x+6} = 2$   
 $x+6 = 8$   
 $x = 2$   
 Check  $\checkmark$   
 $S = \{2\}$

②  $y = -2\sqrt{4-2x} + 3$

$4-2x \geq 0$   
 $4 \geq 2x$   
 $x \leq 2$

$d = \{x \mid x \leq 2, x \in \mathbb{R}\}$

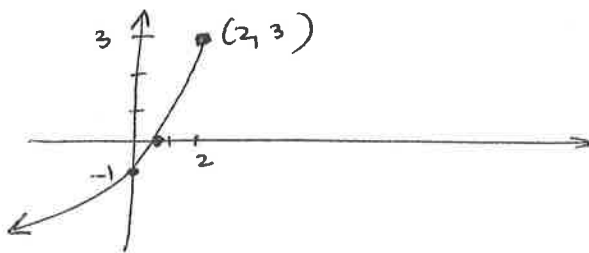
$r = \{y \mid y \leq 3, y \in \mathbb{R}\}$

$x=0 \quad y = -2 \cdot 2 + 3 = -1 \quad (0, -1)$

$y=0 \quad -3 = -2\sqrt{4-2x}$   
 $\left(\frac{3}{2}\right)^2 = 4-2x$

$4 - \frac{9}{4} = 2x \quad \left(\frac{7}{8}, 0\right)$   
 $2x = \frac{7}{4}$

$x = \frac{7}{8}$



③  $y = -2\sqrt{x^2-4} + 1$

$x^2-4 \geq 0 \Rightarrow x \leq -2 \text{ or } x \geq 2$

$d = \{x \mid x \leq -2 \text{ or } x \geq 2, x \in \mathbb{R}\}$

$r = \{y \mid y \leq 1\}$

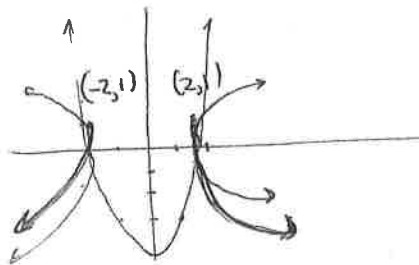
$x=0$  no y-intercept

$y=0 \quad -1 = -2\sqrt{x^2-4}$

$\frac{1}{2} = x^2-4$

$x^2 = \frac{17}{4}$

$x = \pm \frac{\sqrt{17}}{2}$



## Ch. 2 Polynomial Functions

① 5

② e

③ a

④ e

⑤  $y = -\frac{1}{2}(x-6)(x-2)(x+3)$

⑥  $-4, 0, \frac{7}{4}$


⑦ -135

⑧ 3

⑨ -1

⑱  $a < -4$  or  $a > 2$

⑲  $x < 0$  or  $x > 3$

⑳ 

㉑  $-4 < x < 2$  or  $x > 6$

㉒  $x \leq -3$  or  $2 \leq x \leq 5$

⑩ 72

⑪ 24

⑫ 0

⑬  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12,$   
 $\pm \frac{1}{2}, \pm \frac{3}{2}$

⑭ 8

⑮  $-3, \frac{2}{5}, \frac{3}{4}$

⑯  $-\frac{1}{2}, \frac{-3 \pm \sqrt{7}}{2}$

⑰ -2

**Ex. 4**

①  $3^x \cdot 3^{2(2x-1)} = 3^{x+4}$   
 $3x + 4x - 2 = x + 4$   
 $6x = 6$   
 $x = 1$

②  $A = A_0 x^{\frac{t}{T}}$   
 a)  $A = 40 \left(\frac{1}{2}\right)^{\frac{48}{14.9}} = 4.3g$   
 b)  $2.5 = 40 \left(\frac{1}{2}\right)^{\frac{t}{14.9}}$   
 $t = 59.6h$

③ a)  $A(t) = A_0 \left(\frac{1}{4}\right)^{\frac{t}{7}}$   
 b) 64 times  
 c) 3.5 days  
 d) 7 days ago  
 e) 252 days

④ 6      ⑤  $40 = 100 (0.97)^{\frac{t}{5}}$       ⑥  $x = 9^{\frac{1}{2}} = 3$

⑦  $(x+1) \log 5 = \log 2 + 2x \log 3$   
 $x = \frac{\log 2 - \log 5}{\log 5 - 2 \log 3}$

⑧  $\log_a b = 3$

⑨  $-2x + 6 > 0$   
 $x < 3$

⑩  $\frac{\log_4 30}{\log_4 5}$

⑪  $\log x - \log 2 - 3 \log y$

⑫  $3^{-2x} = 3^{3(2-x)}$   
 $-2x = 6 - 3x$   
 $x = 6$

⑬  $x(x-1) = 8$   $x > 0$   
 $x^2 - x - 8 = 0$   $x > 1$   
 $x = \frac{1 \pm \sqrt{33}}{2}$   
 $x = \frac{1 + \sqrt{33}}{2}$

⑭  $y = 3^{x-h} - 4$   
 $x=0 \quad 5 = 3^{-h} - 4$   
 $9 = 3^{-h}$   
 $h = -2$

⑮  $A = 12000 (3)^{\frac{t}{8}}$   
 $\frac{300000}{12000} = \frac{12000}{12000} \cdot 3^{\frac{t}{8}}$   
 $t =$

⑯  $\log \frac{m}{nk^3}$

$y = 3^{x+2} - 4$

⑰  $\begin{cases} 5-x > 0 \\ x-1 > 0 \\ x-1 \neq 1 \end{cases} \rightarrow 1 < x < 5, x \neq 2$

⑱ a

⑲  $A = 100 (5)^{\frac{k}{30}}$

⑳  $x = 3 \cdot 2^{y-2} + 5$   
 $\frac{x-5}{3} = 2^{y-2}$   
 $y-2 = \log_2 \left(\frac{x-5}{3}\right)$

$y = f^{-1}(x) = \log_2 \left(\frac{x-5}{3}\right) + 2$

㉑  $\frac{(x+4)^2}{-x} = 9$   $\begin{cases} x+4 > 0 \\ -x > 0 \end{cases}$   
 $x^2 + 8x + 16 + 9x = 0$

㉒  $(2a)^t = p$

㉓  $\log \frac{a}{b^2 c^3}$

$x^2 + 17x + 16 = 0$   
 $(x+16)(x+1) = 0$   
 $x = -16, x = -1$   
 ↓  
 reject

㉔  $\frac{3x}{x-3} = 25$   
 $3x = 25x - 75$   
 $x = \frac{75}{22}$

㉕  $2x + 4 = 4x - 3 + 5$   
 $x = \frac{2}{2} = 1$

$$(26) \quad 2.6$$

$$(27) \quad 28.20 \text{ days}$$

$$(28) \quad b^x = \frac{c}{a} \\ x = \log_b \frac{c}{a}$$

$$(29) \quad x \log 2 = \log 3 + (6x+1) \log 5$$

$$x = \frac{\log 3 + \log 5}{\log 2 - \log 5}$$

$$(30) \quad 9 - x^2 = 5 \quad \begin{cases} 3-x > 0 \\ 3+x > 0 \end{cases}$$

$$x^2 = 4 \\ x = 2 \text{ or } -2$$

$$(31) \quad 3 + (-1) = \log_4 x$$

$$2 = \log_4 x$$

$$x = 4^2 = 16$$

$$(32) \quad \log_4 (\log_5 x) = \frac{1}{2}$$

$$\log_5 x = 4^{\frac{1}{2}} = \sqrt{4} = 2$$

$$x = 25$$

$$(33) \quad x = 6$$

$$(34) \quad \log_2 \frac{8\sqrt{x}}{y^3}$$

$$(35) \quad \log_{16} x = \frac{\log_4 x}{\log_4 16} = \frac{a}{2}$$

$$(36) \quad \log 2400 = \log (8 \cdot 3 \cdot 100)$$

$$= \log 8 + \log 3 + \log 100$$

$$= 3 \log 2 + \log 3 + 2$$

$$= 3a + b + 2$$

$$(37) \quad a^{\log_a 16} = 16$$

$$(38)$$

$$\log_n a + 2 \log_n b =$$

$$= 5 + 2 \cdot 3 = 11$$

①  $(3 \cos x - 2)(\cos x - 2) = 0$

$\cos x = \frac{2}{3}$        $\cos x = 2$   
no solutions

$x_1 = \cos^{-1}(\frac{2}{3}) = 0.8411$

$x_2 = 2\pi - \cos^{-1}(\frac{2}{3}) = 5.4421$

$0.8411 + 2n\pi$   
 $5.4421 + 2n\pi$ ,  $n$ -integer

③  $l = R\theta_{rad}$

$l = 20 \cdot \frac{3\pi}{4} = 15\pi \text{ cm}$

④ a)  $\sec \frac{4\pi}{3} = -\frac{2}{\sqrt{3}}$

b)  $\tan \frac{7\pi}{6} = -\frac{\sqrt{3}}{3}$

c)  $\sin(-\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}$

⑤ a)  $\sin x = \frac{1}{2}$

$x_1 = \frac{\pi}{6}$   
 $x_2 = \frac{5\pi}{6}$

b)  $\sin 2x = \frac{1}{\sqrt{2}}$

$\sin a = \frac{1}{\sqrt{2}}$   
 $a_1 = \frac{\pi}{4}$   
 $a_2 = \frac{3\pi}{4}$

$x_1 = \frac{\pi}{8}$ ,  $x_2 = \frac{3\pi}{8}$

$x_3 = \frac{9\pi}{8}$ ,  $x_4 = \frac{11\pi}{8}$

⑥  $\sin x = 1 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$  or  $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$        $x = \frac{3\pi}{2}$

⑦  $\tan x (2\cos x - \sqrt{3}) = 0$

$\tan x = 0$        $\cos x = \frac{\sqrt{3}}{2}$

$x_1 = 0$        $x_2 = \frac{\pi}{6}$        $x_3 = -\frac{\pi}{6}$

⑧  $\pi$ ,  $\pi + 6n\pi$ ,  $2\pi + 6n\pi$ ,  $n \in \mathbb{Z}$

⑨  $0.26 + \frac{2n\pi}{3}$   
 $0.79 + \frac{2n\pi}{3}$        $n \in \mathbb{Z}$

⑩  $\frac{7\pi}{6} + 2n\pi$ ,  $\frac{11\pi}{6} + 2n\pi$ ,  $n \in \mathbb{Z}$

⑪  $\frac{\pi}{2} + 2n\pi$ ,  $\frac{3\pi}{2} + 2n\pi$ ,  $\frac{\pi}{4} + 2n\pi$ ,  $\frac{5\pi}{4} + 2n\pi$ ,  $n \in \mathbb{Z}$   
 $\frac{\pi}{2} + n\pi$

⑫  $\begin{cases} \cos \theta \neq \frac{1}{2} \\ \cos \theta \neq 0 \end{cases}$

⑬  $\frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\sin \theta} + \frac{1}{\sin \theta}$   
 $= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} = \sec^2 \theta$

⑭  $3 \cos(x)$

$$(15) \frac{6 \sin \theta}{2 \sin \theta \cos \theta} = \frac{3}{\cos \theta} = 3 \sec \theta$$

$$(17) \sec \theta = -\frac{\sqrt{29}}{2}$$


$$(18) a = 3 \quad y = -3 \cos 2\left(x - \frac{\pi}{6}\right) + 2$$

$$P = \frac{2\pi}{2} = \pi$$

$$\text{Max} = 2 + 3 = 5$$

$$\text{Min} = 2 - 3 = -1$$

## 11. (7) COMBINATORICS

$$(1) 30 C_5$$

$$(2) T_4 = {}_6 C_3 (3x)^3 (-2)^3$$

$$= 20 \cdot 27x^3 \cdot (-8)$$

$$= -4320x^3$$

$$(3) 3360$$

$$(4) 181440$$

$$(5) 3744$$

$$(6) \frac{10!}{7!} = 720$$

$$(7) -80x^2y^3$$

$$(8) n = 7$$

$$(9) \frac{33!}{5!28!}$$

$$(10) T_6 = {}_{10} C_5 (x)^5 (-y)^5$$

$$= -252x^5y^5$$

$$(11) \text{ a) } 4060$$

$$\text{ b) } 24360$$

$$\text{ c) } 1900$$

$$(12) 8$$

$$(13) 210$$

$$(14) 2496$$

$$(15) 60$$

$$(16) 126$$

$$(17) 45$$

$$(18) 201058$$

$$(19) 1728$$