Name: _____ Blk: _____ Date: _____

PreCalculator 12

3.4 Graphing Rational Functions

To Graph Rational Fxns:

1) Find VA's by solving where denominator = 0.

- VA's are where the fxn is undefined, or does not exist.

2) Find HA's by seeing if degree in numerator is >, <, or = to degree in denominator.

- HA's are where the fxn's **ends** approach, but never reaches.

3) Find any x and y intercepts. Also locate any holes.

4) Create a table of values. Be sure to include x values close to the left & close to the right of any VA's.

5) Create a smooth curve through all the points.

Example 1: Graph

a) $f(x)=\dfrac{1}{x-3}+2$

VA $x=3$

HA $y=0+2=2$

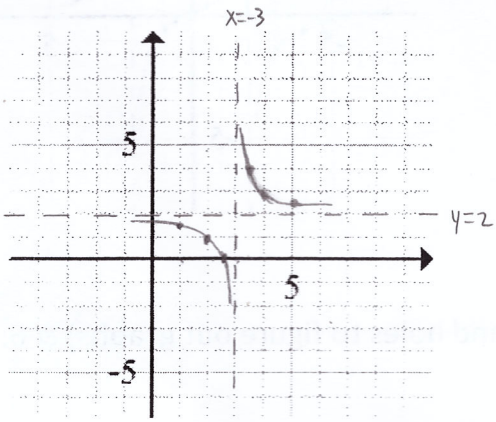
x	$y=\frac{1}{x}$
0	∅
1	1
-1	-1
2	0,5
0,5	2

$(x+3, y+2)$	
3	∅
4	3
2	1
5	2,5
3,5	4

Need a couple more points

$x=1$ $\quad y=\frac{1}{x-3}+2 \longrightarrow$ y 1,5

$x=2.5$ $\longrightarrow$ 0



b) $f(x)=\dfrac{x^2-x-12}{4-x} = \dfrac{(x-4)(x+3)}{-(x-4)}$

Hole @ $x=4$

$f(x)=-(x+3)$

$=-x-3$

$f(4)=-4-3$

$=-7$

hole @ $(4,-7)$

no VA

no HA

$f(0)=-3$

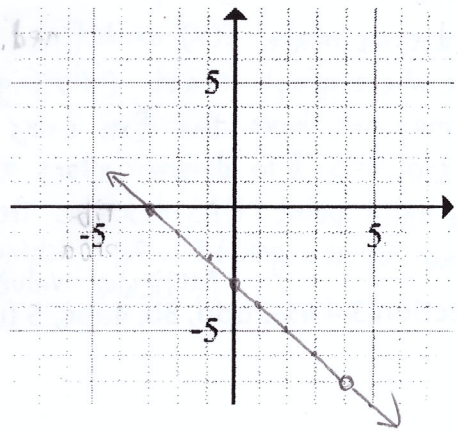
$f(x)=-x-3$

$0=-x-3$

$x=-3$

y-int @ $(0,-3)$

x-int @ $(-3,0)$



c) $f(x)=\dfrac{x^2}{x^2-4}$ $(x+2)(x-2)$

VA @ $x=\pm 2$ no holes

HA $= \frac{1}{1} \rightarrow y=1$
$m=n$

x-int: $y=0$

$0=\dfrac{x^2}{x^2-4}$

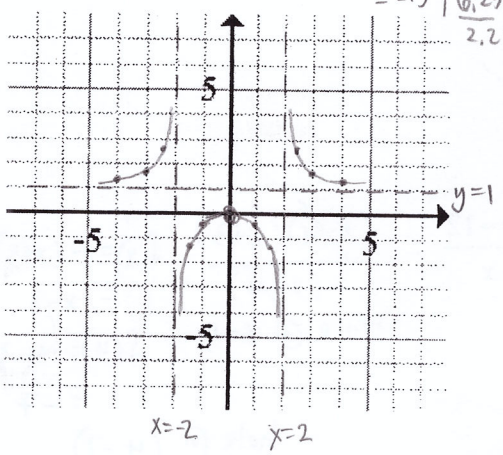
$0=x^2$

$x=0$

$(0,0) \leftarrow$ also y-int

Good ol' TOV

x	$y=\dfrac{x^2}{x^2-4}$
1	$-\frac{1}{3}$
-1	$-\frac{1}{3}$
± 1.5	$\frac{2.25}{-1.75} = -1.3$
± 3	$9/5 = 1.8$
± 4	$16/12 = 1.3$
± 2.5	$\frac{6.25}{2.25} = 2.8$



d) $f(x)=\dfrac{3x^2}{x^2+1}$ no VA

HA $m=n$ so $\frac{3}{1}$

HA @ $y=3$

no holes

x-int: $y=0$

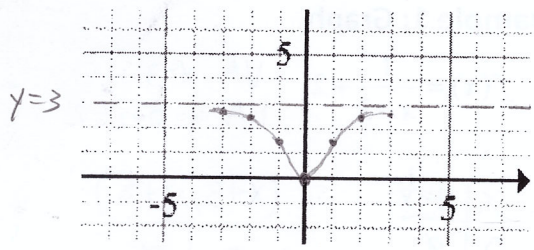
$0=\dfrac{3x^2}{x^2+1}$

$0=3x^2$

$x=0$

$(0,0) \leftarrow$ also y-int

x	$y=\dfrac{3x^2}{x^2+1}$
± 1	$3/2$
± 2	$12/5 = 2.4$
± 3	2.7



e) $f(x)=\dfrac{2x}{x^2-4} = \dfrac{2x}{(x+2)(x-2)}$

no holes VA @ $x=\pm 2$

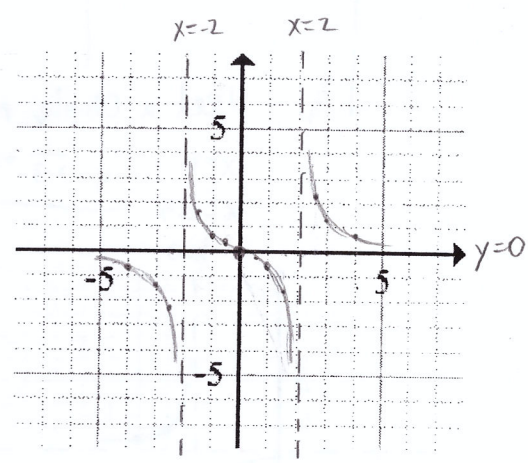
HA: $m<n \rightarrow$ $\boxed{y=0 \text{ HA}}$
$m=1$
$n=2$

x-int: $y=0$

$0=\dfrac{2x}{x^2-4} \rightarrow x=0$ $\boxed{\therefore (0,0) \text{ is x \& y int}}$

Here need enough points to get enough sense of the graph's shape

x	$y=\dfrac{2x}{x^2-4}$
-1	$2/3$
1	$-2/3$
0.5	-0.27
-0.5	0.27
1.5	-1.7
-1.5	1.7
2.5	2.2
-2.5	-2.2
3	1.2
-3	-1.2
4	0.75
-4	-0.75



* Note here (0,0) is a point on the graph b/c it doesn't make f(x) undefined. So while VA's are where graph is not existing, HA's may or may not have the fxn cross it. When $x=0$, $y=0$ & no denominators are $=0$ $\therefore$ it's a valid point. HA's describe the end behaviour of the graphs... approaching a particular value. Here $y=0$.

HW: Section 3.4 #1-2odd, 3b, 4abc, 5 (use VA, HA, intercepts and holes to figure out graphs) & 6.