Name: ____________________ Blk: ________

Date: ______________________

PreClueless 12

3.3 Rational Functions

Rational Functions have the form:

$$f(x) = \frac{g(x)}{h(x)}$$

Ex: $y = \frac{3x^2 - 6x - 1}{x^3}$

Where $g(x)$ and $h(x)$ are polynomials & $h(x)$ $\neq 0$.

1) Asymptote = a line for a curve/fxn whose distance between the line and the curve/fxn approaches zero but never reaches zero. There are 2 types of asymptotes:

a) Vertical **asymptote (VA)** – found whenever denominator = 0.

b) Horizontal **asymptote (HA)** – depends on the degree of numerator & denom:

Consider $f(x) = \frac{g(x)}{h(x)} = \frac{a_m x^m + .. + a_1 x + a_0}{b_n x^n + .. + b_1 x + b_0}$ where $a_m \neq 0,\ b_n \neq 0$

- If $m < n$, HA is $y = 0$.
- If $m = n$, HA is $y = \frac{a_m}{b_n}$ (ratio of the leading coefficients).
- If $m > n$, no HA.

Example 1: Find the VA & HA of

a) $f(x) = -\frac{2x^2 + x + 1}{3x^2 - 12}$

VA: $3x^2 - 12 = 0$

$\frac{3x^2 = 12}{3}$

$x^2 = 4$

$x = \pm 2$

$\boxed{\text{VA @ } x = \pm 2}$

HA: since $m = n$, HA is: $\boxed{y = -\frac{2}{3}}$

b) $f(x) = \frac{2x^3}{(x-1)^2} = \frac{2x^3}{x^2 - 2x + 1}$

VA: $(x-1)^2 = 0$

$x - 1 = 0$

$\boxed{\text{VA @ } x = 1}$

HA: since $m > n$ no HA.

<u>Calculus Method</u>: Divide each term by the highest power of the denominator. Evaluate for $x = \infty$ to find any HA.

Ex a) $f(x) = -\frac{2x^2 + x + 1}{3x^2 - 12}$

$$= -\frac{\frac{2x^2}{3x^2} + \frac{x}{3x^2} + \frac{1}{3x^2}}{\frac{3x^2}{3x^2} - \frac{12}{3x^2}}$$

$$\lim_{x \to \infty} f(x) = -\frac{\frac{2}{3} + 0 + 0}{1 - 0}$$

$\boxed{y = -\frac{2}{3}}$

2) Holes in rational functions AKA **Points of** discontinuity.

- When a VA is Cancelled out through simplification of the fxn.
- Graph will have a hole at this point instead of a VA.

Example 2: Find the x & y intercepts and any holes in each rational function.

a) $f(x)=\dfrac{5}{4x}$

$\boxed{\text{No x or y intercepts}}$ (Unsolvable setting $x=0$ or $y=0$).

$\boxed{VA @ x=0}$

HA: $0<1$ (m, n) $\rightarrow$ HA $\boxed{y=0}$

$\boxed{\text{No holes}}$

b) $f(x)=\dfrac{(x+2)^2}{x^2-4}=\dfrac{(x+2)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$

hole when $x=-2$

$f(x)=\dfrac{x+2}{x-2}$

$\boxed{VA @ x=2}$

$f(-2)=\dfrac{0}{-4}$ $\boxed{\text{hole at } (-2,0)}$

HA: $1=1$ (m, n) $\rightarrow y=1$

$\boxed{HA @ y=1}$

X-int:

$0=\dfrac{x+2}{x-2}$

$0=x+2$

$x=-2$ reject b/c there's a hole at $x=-2$

y-int:

$y=\dfrac{x+2}{x-2}$

$=\dfrac{0+2}{0-2}$

$=-1$

$\boxed{\text{y-int } (0,-1)}$

c) $y=\dfrac{x^2+2x-8}{x^3-4x}=\dfrac{(x+4)(x-2)}{x(x^2-4)}=\dfrac{(x+4)\cancel{(x-2)}}{x(x+2)\cancel{(x-2)}}$

hole at $x=2$

$f(2)=\dfrac{2+4}{2(2+2)}=\dfrac{6}{8}=\dfrac{3}{4}$

$\boxed{\text{hole at } (2,\tfrac{3}{4})}$

$\boxed{VA: x=0,\ x=-2}$

HA: $m<n$, $1<2$ $\rightarrow y=0$

$\boxed{HA @ y=0}$

y-int: $x=0$

$y=\dfrac{0+4}{0(0+2)}=\dfrac{4}{0}$

$\boxed{\text{no y-int}}$

x-int: $y=0$

$0=\dfrac{x+4}{x(x+2)}$ $\rightarrow 0=x+4$, $x=-4$

$\boxed{(-4,0)}$

d) $y=\dfrac{2}{x}-8$

$\boxed{\text{no holes}}$

$\boxed{VA: x=0}$

HA: $0<1$ (m, n) $\rightarrow y=0$ but graph's shifted down 8 so $\boxed{HA @ y=-8}$

x-int: $y=0$

$0=\dfrac{2}{x}-8$

$\dfrac{8}{1}=\dfrac{2}{x}$

$x=\dfrac{2}{8}=\dfrac{1}{4}$

$\boxed{(\tfrac{1}{4},0)}$

y-int: $x=0$

$\varnothing$

$\boxed{\text{no y-int}}$

e) $f(x)=\dfrac{x^2-2x-15}{x-5}=\dfrac{\cancel{(x-5)}(x+3)}{\cancel{(x-5)}}$

$f(x)=x+3$

Hole @ $x=5$

$f(5)=5+3$

$=8$

$\boxed{\text{hole @ } (5,8)}$

$\boxed{VA: \text{none}}$

HA: $1>0$ (m, n) $\rightarrow$ no HA

$\boxed{\text{no HA}}$

X-int: $y=0$

$0=x+3$

$x=-3$

$\boxed{(-3,0)}$

y-int: $x=0$

$y=x+3$

$=0+3$

$y=3$

$\boxed{(0,3)}$

f) $f(x)=\dfrac{3x+1}{4x-2}-1$

VA: $4x-2=0$

$4x=2$

$x=\dfrac{1}{2}$

$\boxed{VA @ x=\tfrac{1}{2}}$

$\boxed{\text{No holes}}$

HA: $m=n$ so $y=\dfrac{3}{4}$ but graph shifted down 1

so $y=\dfrac{3}{4}-1=-\dfrac{1}{4}$

$\boxed{HA @ y=-\tfrac{1}{4}}$

X-int: $y=0$

$0=\dfrac{3x+1}{4x-2}-1$

$\dfrac{1}{1}=\dfrac{3x+1}{4x-2}$

$3x+1=4x-2$

$3=x$

$\boxed{(3,0)}$

y-int: $x=0$

$y=\dfrac{0+1}{0-2}-1$

$=-\dfrac{1}{2}-1$

$=-\dfrac{3}{2}$

$\boxed{(0,-\tfrac{3}{2})}$

HW: Section 3.3 # 2-4 odd letters