With regards to applications, composition is used anytime a change in one quantity produces a change in another, which, in turn, produces a change in a third quantity. Example: The _____ of travelling by car depends on the amount of _____ used, and the amount of _____ used depends on the number of _____driven.

A few caveats:

1) It is entirely possible to do composite functions with itself:

2) Composite functions multiplication functions! This is a common mistake. Beware:

3) Composition is not commutative:

$\boxed{x \neq 0}$

Example 3: Consider $f(x) = \sqrt{x-2}$, $g(x) = x^2$ and $h(x) = \frac{1}{x}$. Determine the following and find the domain, where necessary:

a) $(f \circ g)(2)$

b) $(g \circ f)(1)$

c) $(f \circ f)(6)$

d) $f \circ g \circ h$

$g(x) = x^2$

$g(h(x)) = \left(\frac{1}{x}\right)^2 = \frac{1}{x^2}$

$f(x) = \sqrt{x-2}$

$f(g(h(x))) = \sqrt{\left(\frac{1}{x^2}\right) - 2} = \sqrt{\frac{1}{x^2} - \frac{2x^2}{x^2}} = \boxed{\frac{\sqrt{1-2x^2}}{x}}$

$x \neq 0$

Another restriction: $1 - 2x^2 \geq 0$

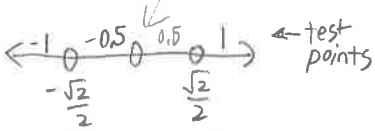
$1 \geq 2x^2$

$\frac{1}{2} \geq x^2$

$\frac{1}{2} = x^2$

$\pm\sqrt{\frac{1}{2}} = x$

$\pm \frac{\sqrt{2}}{2} = x$



For $x = -1$ $\frac{1}{2} \geq x^2$

$\frac{1}{2} \geq 1$ NO!

$x = -0.5$ $\frac{1}{2} \geq (-0.5)^2 = \frac{1}{4}$ YES

$x = 0.5$ $\frac{1}{2} \geq (0.5)^2 = \frac{1}{4}$ YES

$x = 1$ $\frac{1}{2} \geq (1)^2 = 1$ NO

$\therefore D\{x \mid -\frac{\sqrt{2}}{2} \leq x < 0,\ 0 < x \leq \frac{\sqrt{2}}{2},\ x \in \mathbb{R}\}$

Example 4: Given $f(x) = x + 1$ and $g(x) = \frac{1}{x-1}$, determine the following and find the domain:

a) $(f \circ g)(x)$

$f(x) = x + 1$

$f(g(x)) = \frac{1}{x-1} + 1$

$= \frac{1}{x-1} + \frac{x-1}{x-1}$

$= \frac{x}{x-1}$

$g(x)$ domain $x \neq 1$

also $(f \circ g)(x)$ $x \neq 1$.

$\therefore D\{x \mid x \neq 1,\ x \in \mathbb{R}\}$

b) $(g \circ f)(x)$

$g(x) = \frac{1}{x-1}$

$g(f(x)) = \frac{1}{(x+1)-1}$

$= \frac{1}{x}$

$f(x)$ domain no restrictions

but $g \circ f$ $x \neq 0$

$\therefore D\{x \mid x \neq 0,\ x \in \mathbb{R}\}$

c) $(g \circ g)(x)$

$g(x) = \frac{1}{x-1}$

$g(g(x)) = \frac{1}{\left(\frac{1}{x-1}\right) - 1} = \frac{1}{\frac{1}{x-1} - \frac{x-1}{x-1}} = \frac{1}{\frac{1-x+1}{x-1}}$

$= \frac{1}{\frac{2-x}{x-1}} = 1 \div \frac{2-x}{x-1} = 1 \times \frac{x-1}{2-x}$

$= \frac{x-1}{2-x}$

$g(x)$ domain $x \neq 1$

$g(g(x))$ domain $x \neq 2$

$\therefore D\{x \mid x \neq 1, 2,\ x \in \mathbb{R}\}$