Week 3: Discriminant and Vieta’s theorem

Discriminant: $∆=b^{2}-4ac$

For question 1 to 3, find the discriminant and determine the number and the kind of roots of each equation.

1. $2x^{2}-x-3=0$
2. $\frac{3x-2}{2x-1}=\frac{1}{3-x}$
3. $-\sqrt{2}x^{2}+3x+2\sqrt{x}=0$

For question 4 to 6, find the value of k so that the equation has the indicated number of solutions.

1. $kx^{2}+x+k=0$, two real solutions
2. $x^{2}-kx+4=0$, one real solution
3. $x^{2}-kx+4=0$, no real solution

Advanced

1. $x^{2}+7x+c=0$ has real solutions. If c is an integer, then what is the greatest possible value of c?
2. If $2x^{2}+2\sqrt{2}x+tanα=0$ has equal roots, and $0°<a<180°$, then $a=\left( \right).$
3. 45° B. 135° C. 45° or 135° D. 30°
4. Real numbers a and b satisfy the equation: $a^{2}b^{2}+a^{2}+6ab+2a+9=0$. Then $b\geq ( )$.

Vieta’s Theorem:

$ax^{2}+bx+c=0$ has two solutions $x\_{1}$ and $x\_{2}$.

$$x\_{1}+x\_{2}=-\frac{b}{a}$$

$$x\_{1}×x\_{2}=\frac{c}{a}$$

1. Find $x\_{1}+x\_{2}$ of this equation: $x^{2}-2x-3=0$
2. Find $x\_{1}x\_{2}$ of this equation: $5x^{2}+21x-5=0$
3. Find the $x\_{1}+x\_{2}$ and the $x\_{1}x\_{2}$ of this equation: $2x^{2}+6x+7=0$
4. Find $\frac{1}{x\_{1}}+\frac{1}{x\_{2}}$ of this equation: $8x^{2}+5\sqrt{2}x+7=0$

Hint: $\frac{1}{x\_{1}}+\frac{1}{x\_{2}}=\frac{x\_{1}+x\_{2}}{x\_{1}x\_{2}}$

1. Find $x\_{1}^{3}+x\_{2}^{3}$ of this equation: $-2x^{2}+9x-6=0$

Hint: $x\_{1}^{3}+x\_{2}^{3}=(x\_{1}+x\_{2})(x\_{1}^{2}-x\_{1}x\_{2}+x\_{2}^{2})$

Advanced

1. Let the two roots of $x^{2}+px+q=0$ be $a$ and $b$. Set $I\_{1}=a+b$, $I\_{2}=a^{2}+b^{2}$, ……, $I\_{n}=a^{n}+b^{n}$. When $n\geq 3$, find the value of $I\_{n}+pI\_{n-1}+qI\_{n-2}.$
2. Let the two roots of $x^{2}-x-1=0$ be $a$ and $b$.Find the value of $a^{4}+3b$.
3. Let the two roots of $x^{2}-13x+m=0$ be $p$ and $q$. If $p$ and $q$ are prime numbers, then what is $\frac{p}{q}+\frac{q}{p}$?
4. If $a×b\ne 1$, and $5a^{2}+2013a+9=0$, $9b^{2}+2013b+5=0$, then what is $\frac{a}{b}$?
5. For natural number $n$, let the two roots of $x^{2}+\left(2n+1\right)x+n^{2}=0$ be $a\_{n}$ and $b\_{n}$, find the value of this equation:

$$\frac{1}{(a\_{3}+1)(b\_{3}+1)}+\frac{1}{(a\_{4}+1)(b\_{4}+1)}+\frac{1}{(a\_{5}+1)(b\_{5}+1)}+…+\frac{1}{(a\_{20}+1)(b\_{20}+1)}$$